Fractal Nature of Superconducting Percolating Cluster

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---ABSTRACT--

The percolative behaviour and fractal nature of 123-ortho I phase of YBCO system has been studied in detail by Monte Carlo simulation technique. In a 2D-square lattice (Cu-O-Cu chains), for different lattice dimensions from 60×60 up to 150×150 the simulation study has been carried out. It has been found that for almost all lattice dimensions, $YBa_2Cu_3O_{7-\delta}$ - system for $\delta > 0.4$ leads to tetragonal phase; for $0.35 < \delta < 0.40$ phase transition from tetragonal to orthorhombic begins; For $0.23 < \delta < 0.35$ a weak link behaviour has been observed, and this value is in good agreement with earlier reported works . For $0.01 < \delta < 0.23$ a strong percolative behaviour has been observed indicating the single Ortho I phase. By increasing the periodicity of the random number generator used, a too sharp phase transition curves has been observed, exhibiting real phase transition as observed experimentally. Then by applying the scaling laws on a superconducting percolating cluster near its critical region (at $P \geq Pc$), two basic critical exponents β and γ has been determined and is used to determine Fractal dimensionality of the superconducting percolating cluster. This value is found to be in good agreement with the earlier reported works and is seems to be constant for different lattice dimension. It indicates that the fractal dimensionality does not change by different lattice dimension but can vary with lattice type.

Keywords - Percolation, Phase transition, Fractal dimension.

I. INTRODUCTION

The study of fractal dimension could be applied in disposing irregular problems which is beyond the concept of traditional integral dimension as perceived by the Euclidean Geometric concepts. The fractal parameter that reflects the self-similarity is Fractal dimension. Fractal nature of the 90K superconductive percolating cluster of YBCO system has been presented in this present paper. Aggregation of particles often produces fractal clusters. Many experiments and numerical simulations have recently explored the properties of aggregation kinetics, gelation and sedimentation, with emphasis on their fractal structure. Ideal fractals are self-similar on all length scales, and therefore all their properties follow power laws in their linear scale, L; there exists no basic length which can be used as unit of length. In physical problems, there always exists a smallest length, e.g., linear size of one particle. The fractal dimension Dwas first noticed by Stanley [1], the structure of percolation clu clusters can be well described by the fractal concepts [2]. The Fractal Cluster Model has been used in the Superconducting Glasses context first by Smith, Binder [3] and Abrikosov [4] and it has been successively developed, in a certain number of papers, by Malozemoff and Barbara and Continentino and Malozemoff [5]. Thus with this frame work of Fractal Cluster Model for the granular high Tc superconductors, which falls within the inhomogeneous limit, the fractal dimensionality of the material can be calculated. This inhomogeneous limit makes us possible to

calculate the fractal dimensionality of the granular high TC Superconducting materials by studying the infinite percolation clusters near the critical region.

II. THEORY

The main assumption of the FCM is the existence of a characteristics distribution of a cluster size [5] and is given by

$$n_s = s^{-\tau} f_c \frac{s}{S_\xi} \tag{1}$$

Where n_s is the number of the S-clusters, and S is the cluster size containing S frozen spins, and $\tau = 2 + \delta^{-1}$ with δ the standard critical exponent; f_s is a function that tends for p close to P_c, to a constant, i.e. S small compared with S_ξ, S_ξ is the typical cluster size that controls the critical behavior at the percolation threshold. It diverges as $|P-P_c|^{-1/\sigma}$ and is related to the correlation length ξ and to the fractal dimension of the system by $S_{\xi \propto} \xi^D$. Two relevant hypothesis of the model are the spins inside a cluster are rigidly clamped and the interaction between clusters is neglected.

In SuG, below the critical temperature, but in the critical region $0.027T/T_c$ it is possible to identify a percolating cluster like for SG. A percolating cluster is constituted by those grains that are connected by Josephson Junction (not necessarily all are coherent in phase) having different J_c. Increasing the temperature for coupling between grains, weakens until, slightly above the critical temperature for percolation. There remain only isolated clusters of connected grains. Equivalently the critical transition is provoked by the growing the dimension of the area occupied by uncoupled clusters so that n_s distribution is defined and a characteristic S_ξ cluster whose divergence controls the critical behaviors close to P_c . This situation is very close to that in SG and pushes us to discuss the consequence of the application of the FCM to the SuG case. From the experimental side the only constraint to respect, in-homogeneously penetrated, and low edge out that the system can be still considered fractal [6]. i.e. that the junction system has not broken. Two important relations have been obtained from the FCM. They are the relation between the fractal dimension D and the crossover exponent ϕ and the relation between ϕ and the exponent ψ defined, by the scaling hypothesis.

In zero field ξ diverge as and thus $|t|^{-\gamma D}$, or in the percolation language it diverges as $S_\xi \sim |p-p_c|^{-1/\sigma}$ so that identifying $|P-P_c|$ with the reduced temperature t, we get $\nu D = 1/\sigma$. The relation between σ, τ and the standard critical ones allow then to show that $\nu D = \phi$. In fact, from [7]

$$\left. \begin{array}{l} 2 - \alpha = (\tau - 1)\sigma \\[4pt] \beta = \dfrac{\tau - 2}{\sigma} \\[4pt] -\gamma = \dfrac{\tau - 3}{\sigma} \\[4pt] \dfrac{1}{\delta} = \tau - 2 \end{array} \right\} \tag{2}$$

It is easy to see that

$$\beta + \gamma = \emptyset = \frac{1}{\sigma} = \vartheta D$$

From (2) it is possible to work out all the usual relations between standard critical exponents and to demonstrate [8] the hyper-scaling law

$$2 - \alpha = \vartheta \delta = \gamma + 2\beta \tag{3}$$

is satisfied; δ is the Euclidean dimensions of the system and in our case always equal to 2. To work out the relation between ϕ and ψ it is necessary to introduce the magnetization in a fractal form

$$M = \frac{1}{p}\Sigma_s S^\gamma R_s \tan h \left(\frac{\mu S^\gamma H}{kT}\right) \tag{4}$$

and to assume that for SuG, like for SG, y= ½ i.e., in the infinite connected cluster because of frustration (even with a J_c) on the average different from zero, the phase is distributed at random and clamped in sub-clusters. Differentiating equation (4) with respect to the field and following the indications of Stauffer and [9] it is easy to derive the divergence of susceptibility by the following expressions:

$$\chi_{DC} = |t|^{-\gamma} f\left(\frac{H}{|t|^{\frac{\beta\delta}{2}}}\right)$$

or in percolation language,

$$\chi_{DC} = |P - P_c|^{-\gamma} f\left(\frac{H}{|P-P_c|^{\frac{\beta\delta}{2}}}\right) \tag{5}$$

using equation (3) we get, finally,

$$2\psi = \emptyset = \vartheta D \tag{6}$$

where D is the fractal dimension of the system considered. Using the standard relations between critical exponents, we can eliminate ν from equation (3) and equation (6) and obtain the relationship:

$$\frac{\beta+\gamma}{2\beta+\gamma} = \frac{D}{d} \tag{7}$$

III. COMPUTATIONAL METHOD

In this present work, the simulation experiment has been carried out in two parts.

Simulation Procedure Part I:

Form a 2D square lattice of dimension   60 x 60.Using Random number generator, between 0.0 and 1.0 has been assigned to each elements of the lattice.By a variable "P", the concentration of occupied oxygen sites in the given lattice is predetermined. Compare the P value with lattice site's value, if lattice site value   ≤ P, assign as occupied site else declare as vacant site.Convert occupied sites as "One's" and vacant sites as zero's.Apply the superconducting condition for Ortho I phase ie., eliminate the oxygen atoms in the oxygen deficient portions of Ortho I phase which is not participate in the ordering. Check the nearest neighbours with the above conditions for continuous connectedness.If continuous connectedness is available, say the percolation is set in for this particular P value.Repeat the above steps for several simulation runs say 100 times.Out of 100 runs, note down the number of times percolation detected.Calculate P_c with the above values as per [12].Repeat the above procedures for different lattice dimension.Obtain various P_c values for different lattice dimension.

Simulation Procedure Part II:

Form the cluster distribution pattern. Try to from the cluster as per the Reference [9]. Obtain the cluster distribution pattern for the given lattice dimension at $P \geq P_c$, near the critical region.Obtain the values of S - cluster and n_s – cluster from the distribution pattern.Apply the scaling laws to obtain two free exponents τ and σ [8].By using the relation among critical exponents in eqn. (2), calculate the values of critical exponents βandγ.Determine the fractal dimensionality using the relation in eqn. (7).Repeat this procedure for various lattice dimensions.

IV. RESULTS AND DISCUSSIONS

In this present work, the simulation experiment has been carried out in two parts. Part I describes the determination of percolation threshold or critical site occupancy for oxygen atoms in the Cu (1) plane.Part II describes the determination of critical exponents and thereby the fractal dimensionality of the superconductive percolating clusters.

Part I:

Number of times percolation detected for various oxygen concentration ie., $0.0 \leq p \leq 1.00$, for 10 simulation runs and for lattice dimension 20x20, 100x100 and 150x150 has been observed and results are represented graphically in fig.1. From this figure, an analogy between phase transition and percolation has been observed. A double phase transition is observed due to poor quality of the random number generator. The effect of periodicity on this behavior is studied. By using the random number generator, whose periodicity 10^9, it is observed that sharp transition has been observed for larger lattice dimensions as observed experimentally and is shown in fig.2.The percolation threshold P_c for different lattice dimension has been calculated and is presented in fig.3.

From the figure it is observed that the P_c values statistically fluctuates more for low lattice dimension and less fluctuates for higher lattice dimension. It is also observed that P_c value seems to approach a certain value. Theoretically P_c has exact value only when number of simulation runs tends to infinite and also for infinite lattice dimension. The value of $P_c = 0.771$ for 500X500 lattice is in good agreement with earlier reported work [11]. True percolation threshold $P_c(\infty)$ has been calculated and is also shown in fig.3. From this figure, it is observed that nearly 76.6 % of oxygen atom should be occupied on the Cu(1) plane in 90K YBCO system. The simulation experiment has also carried out for different simulation runs ie; from 200 up to 1000 in steps of 200 runs and its $P_c(N)$ values are calculated.
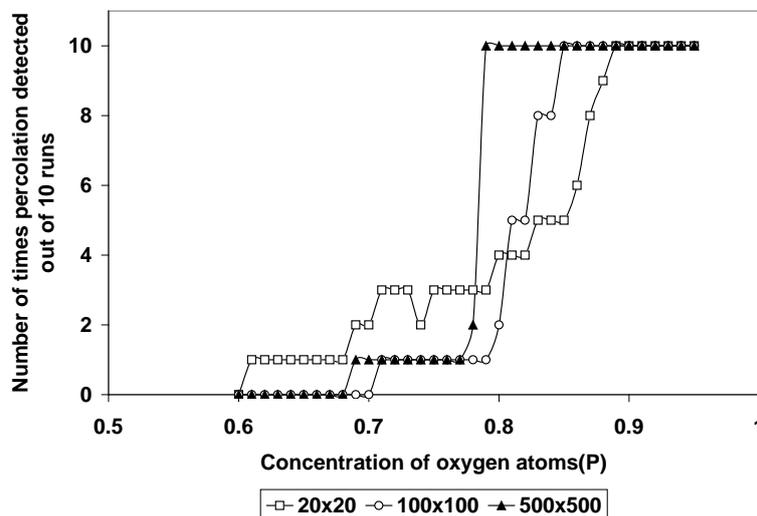

Figure 1: Shows the variation of number of times percolation detected versus the concentration of oxygen atoms.
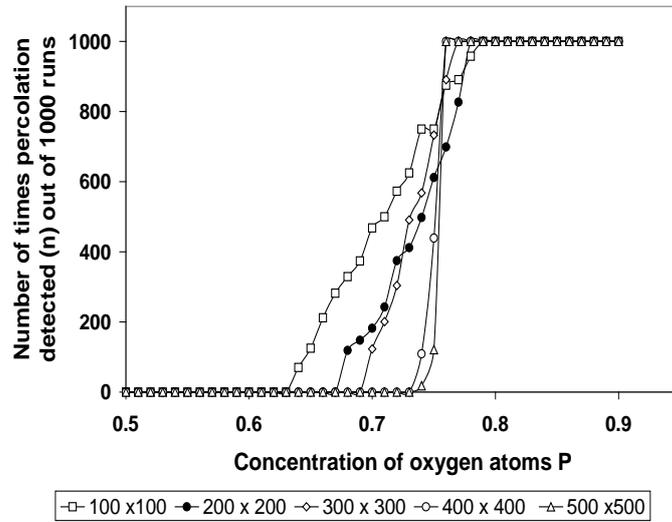
Figure 2: The variation of number of times percolation detected versus the concentration of oxygen atoms by using improved random no. generator (Periodicity 109).
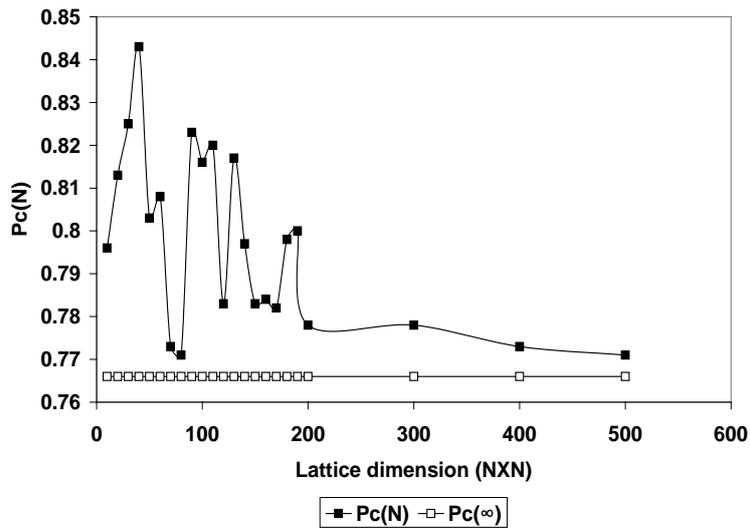

Figure 3: Statistical fluctuations of Pc(N) over Pc(∞).

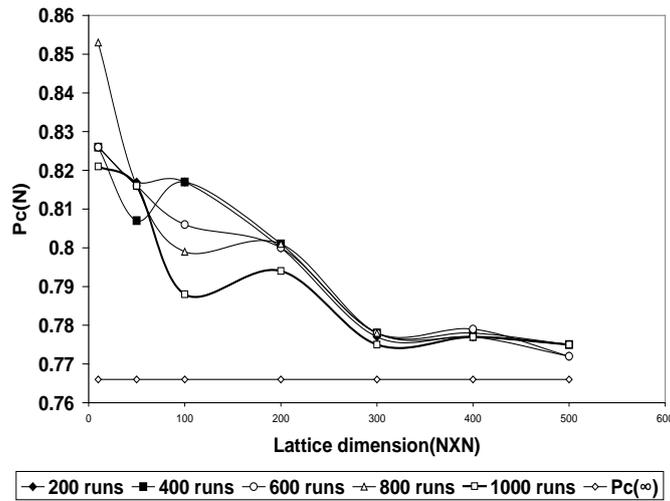Figure 3: Plot of $P_c(N)$ versus different lattice dimension, for different simulation runs.

Figure 4: Statistical fluctuations of Pc(N) over Pc(∞) for different lattice dimensions.

Part II:

Part II of the simulation procedure has been carried to study the cluster behavior near the critical region. The variation of number of clusters with concentration of occupied sites is shown in fig.5. A typical infinite percolating cluster is also shown in fig.6. From fig.5 it is also observed that as P value increases the size of the cluster increases and number of S – cluster decreases. When P < Pc there are many isolated clusters, when P>Pc an infinite cluster will be formed (fig.6).
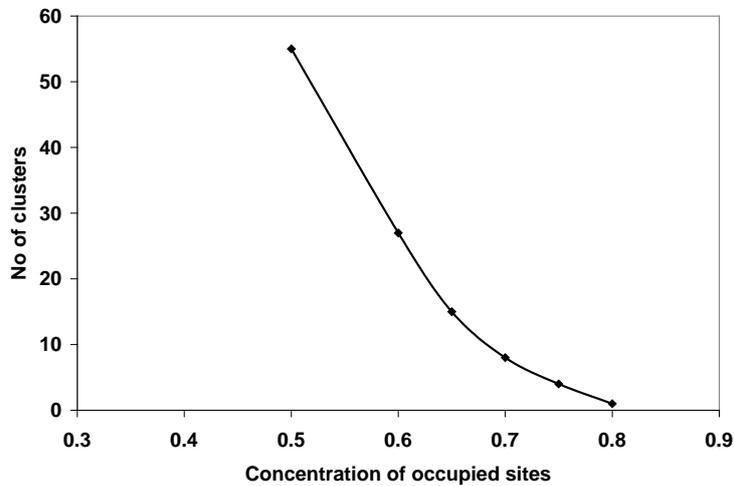

Figure 5: Shows the variation of number of clusters versus concentration of oxygen atoms on the Cu(1) plane.
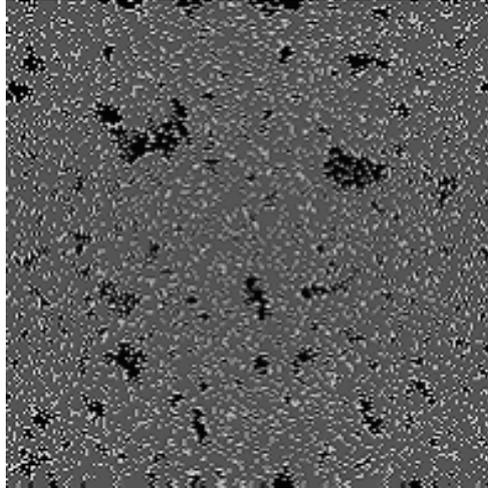
Figure 6: Shows one infinite superconducting percolating clusters for Ortho I phase. Here black color indicates the isolated occupied sites, white indicates the vacant sites & the gray indicates the continuously connected superconducting sites.
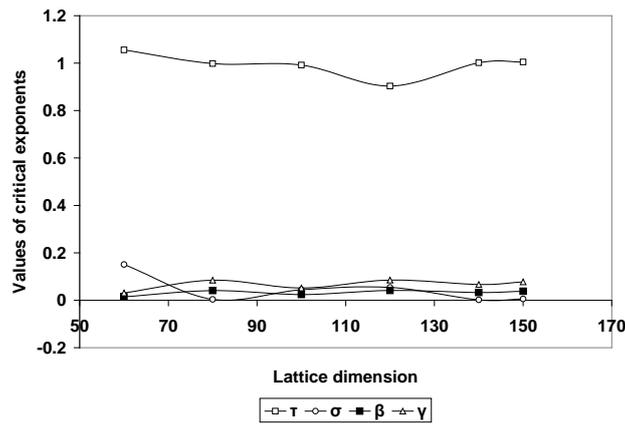

Figure7: Shows the variation of critical exponents with lattice dimension

The scaling laws are applied at the critical point and the values of different critical exponent were determined by using the following relations [8]:

$$n_s(P) \propto S^{-\tau} f(Z) \tag{8}$$
$$\Sigma_s n_s(P) \propto \|P - P_c\|^{(\tau-1)/\sigma} \tag{9}$$

Values of critical exponents τ and σ are calculated for various lattice dimensions from 60X60 upto 150X150 in steps of 20X20. By using the relation among the critical exponents (eq. 2) α and β values are calculated. These values are represented in graphically in fig.7.

From the graph we observed that the critical exponent values are not much varying while increasing the lattice dimension. Basically critical exponents are with respect to lattice type rather than the lattice dimensions. The constant behavior of the critical exponents indicates the universality nature. The values are slightly varying with that of Stauffer. In the present work, we have chosen the super conducting cluster rather than the normally conducting lattices, may be the reason for the variation. It is also observed that the Pc(N) values considerably shifted to 0.766 than 0.593(Stauffer's). The

fractal dimension D for various lattice dimensions is determined both theoretically and by simulation experiment. For theoretical calculation the following relation is used.

$$D = (d - \beta)/\vartheta \tag{10}$$

where d = 2, the Euclidean dimension, and ν = 1.059 is used for the calculation. In the case of simulated values of fractal dimension equation (7) is used for the calculation. All the values seem to be in good agreement with the reported value [13]. Again the fractal dimensionality values seemto constant exhibiting the universality behavior.
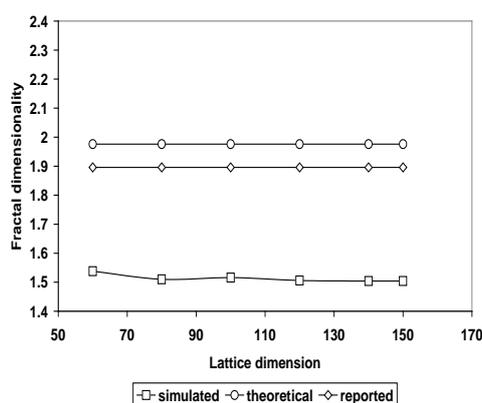

Figure 8: The variation of fractal dimensionality with lattice dimension

V. CONCLUSION

Fractal parameter that reflects the self-similarity is fractal dimension and is determined for superconducting percolating cluster of Ortho I phase. Fractal dimensionality also reflects the universality nature. The value is very good agreement with earlier reported work. Fractal dimensional calculation for other lattice types is kept as the future scope of the work.

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